

Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE Mathematics
Decision D2 Paper 6690_01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark
  ⌙

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1. (a)</td><td>The solution would otherwise be degenerate
B3</td><td>B1
B1 (2)</td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th></th><th></th><th>24
(1)</th><th>27
(2)</th><th>19
(3)</th><th>21
(4)</th></tr>
<tr><td>0</td><td>(A)</td><td>X</td><td>5</td><td>2</td><td>13</td></tr>
<tr><td>4</td><td>(B)</td><td>X</td><td>X</td><td>6</td><td>12</td></tr>
<tr><td>14</td><td>(C)</td><td>–13</td><td>X</td><td>X</td><td>X</td></tr>
<tr><td>15</td><td>(D)</td><td>–16</td><td>–10</td><td>–3</td><td>X</td></tr>
</table>
</td><td>M1 A1 A1 (3)</td></tr>
<tr><td>(c)</td><td>Route is e.g. D1 – B1 – B2 – C2 – C4 – D4
Entering cell is D1, Exiting cell is C2</td><td>M1
A1 (2)</td></tr>
<tr><td></td><td></td><td>7 marks</td></tr>
</table>

Notes for Question 1

a1B1: ‘degenerate’ or an argument based on $n + m - 1$ required (values do not need to be substituted) – not just ‘need 7 values’, ‘there are only 6 entries’, ‘so we can do a stepping-stone method’ or ‘demand and supply has been met’
a2B1: CAO (B3) – could be seen in a diagram (but must be clear)

b1M1: Finding 8 shadow costs and 9 IIs
b1A1: CAO for shadow costs [Alt: A(24), B(28), C(38), D(39), 1(0), 2(3), 3(-5), 4(-3)]
b2A1: CAO for improvement indices – must not be stated as part of a table of costs unless clearly differentiated from these costs e.g. circled, underlined, etc.

c1M1: A valid route (possibly drawn so need not be explicitly stated), their most negative II chosen, only one empty square, θ's balanced
c1A1: Correct stepping-stone route stated or clearly shown on a diagram and CAO for entering and exiting cells

SC1: Those candidates who have a 0 in B3 can score (b) M1A0A0 (c) M1A1 – the M in (b) is for finding 8 shadow costs and 9 IIs. The M mark in (c) is for the correct route D1 – B1 – B3 – C3 – C4 – D4 and the A mark is for the correct entering cell (D1) and correct exiting cell (B1)

SC2: Those candidates who have a 0 in C2 for (b) but then move it to B3 for (c) can score full marks in (b) and then M1 only in (c) for a valid balancing route

Question Number	Scheme	Marks
2. (a)	Row minimum {–3, –5, –2, –3} Row maximin = –2 Column maximum {2,5,5,4} Column minimax = 2 So play safe for player A is 3 and play safe for player B is 1	M1 A1 A1 **(3)**
(b)	$2 \neq -2$ so game is not stable	B1 **(1)**
(c)	Column 1 dominates column 4 because –3 < –1, –5 < –1, –2 < 2 and 2 < 4 (so remove column 4) Row 3 dominates row 2 because –2 > –5, 5 > 3, 4 > 1 (and 2 > –1) (so remove row 2)	B1 B1 **(2)**
(d)	e.g. add at least 3 to each element e.g. $\begin{pmatrix} 1 & 6 & 9 \\ 2 & 9 & 8 \\ 6 & 1 & 3 \end{pmatrix}$, $\begin{pmatrix} 0 & 5 & 8 \\ 1 & 8 & 7 \\ 5 & 0 & 2 \end{pmatrix}$, etc. Let p_1, p_3, p_4 be the <u>probability</u> of (A) playing 1, 3, 4 respectively (where $p_1, p_3, p_4 \geq 0$) Let $V =$ <u>value</u> of the <u>game</u> (to player A) Maximise $(P =)V$ Subject to: e.g. $V - p_1 - 2p_3 - 6p_4 \leq 0$ $\quad V - p_3 - 5p_4 \leq 0$ $V - 6p_1 - 9p_3 - p_4 \leq 0$ $\quad V - 5p_1 - 8p_3 \leq 0$ $V - 9p_1 - 8p_3 - 3p_4 \leq 0$ $\quad V - 8p_1 - 7p_3 - 2p_4 \leq 0$ $p_1 + p_3 + p_4 \leq 1$ $\quad p_1 + p_3 + p_4 \leq 1$	B1 B1 B1 B1 M1 A1 A1 **(7)**
		13 marks

Notes for Question 2

a1M1: Clear attempt to find the Row maximin and Column minimax (either the Row minimums or Column maximums correct or at least six (of the eight) values stated correctly) – if they reduce the game to the given 3 by 3 matrix then they need five of the six values correct – working must be done in (a) – if correct play-safes for both players stated with no working then M1 only
a1A1: Correct Row maximin and Column minimax (dependent on all row mins and column maxs correct) – stated or clearly shown
a2A1: Correct play safes (A(3) and B(1)) for both players – not dependent on previous A mark

b1B1: CAO (dependent on the first two marks in (a) or complete method seen in (b)) – states $2 \neq -2$ or row(maximin) $\neq$ col(minimax) as long as 2 and –2 are clearly identified + conclusion (not stable)

c1B1: Either correct domination stated for both rows and columns **or** one correct with correct justification
c2B1: CSO (correct domination and full justification) – for justification the minimum we will accept is that <u>all values</u> in C4 > C1 (although the C1 could be implied) and that all values in R3 > R2 (again R2 could be implied) – note strict inequalities
d1B1: Making all terms non-negative (any addition of at least 3 is acceptable) – must be shown (not just stated as a method), all values correct – could be implied in constraints
d2B1: Defining probability variables (allow throughout if defined using 1, 2 and 3 rather than 1, 3 and 4)
d3B1: Defining V
d4B1: Maximising + function/expression
d1M1: At least three inequalities (of the four) using column values in (V), p_1, p_3, p_4 (or equations using slack variables e.g. $V - p_1 - 2p_3 - 6p_4 + r = 0$ but must be + r), all probability terms in the first three constraints having correct signs for their coefficients, must be $V \leq ...$
d1A1: The three inequalities in V, p_1, p_3, p_4 CAO (must be expressed as inequalities)
d2A1: Probability sum <u>inequality</u> correct

If defining for the original 4 by 4 game (they would need to add at least 5) then first five marks available only in (d)

Question Number	Scheme	Marks
3. (a)	Reducing rows and columns to get $\begin{pmatrix} 0 & 6 & 2 & 4 & 10 \\ 0 & 4 & 11 & 6 & 8 \\ 0 & 1 & 7 & 8 & 9 \\ 0 & 1 & 2 & x-34 & 7 \\ 5 & 4 & 1 & 0 & 2 \end{pmatrix}$	M1
	then $\begin{pmatrix} 0 & 5 & 1 & 4 & 8 \\ 0 & 3 & 10 & 6 & 6 \\ 0 & 0 & 6 & 8 & 7 \\ 0 & 0 & 1 & x-34 & 5 \\ 5 & 3 & 0 & 0 & 0 \end{pmatrix}$	A1
	Using three lines and augment by 1 to get $\begin{pmatrix} 0 & 5 & 0 & 3 & 7 \\ 0 & 3 & 9 & 5 & 5 \\ 0 & 0 & 5 & 7 & 6 \\ 0 & 0 & 0 & x-35 & 4 \\ 6 & 4 & 0 & 0 & 0 \end{pmatrix}$	M1 A1
	Using four lines and augment by 3 to get $\begin{pmatrix} 0 & 5 & 0 & 0 & 4 \\ 0 & 3 & 9 & 2 & 2 \\ 0 & 0 & 5 & 4 & 3 \\ 0 & 0 & 0 & x-38 & 1 \\ 9 & 7 & 3 & 0 & 0 \end{pmatrix}$	M1 A1ft A1
	So A = 4, B = 1, C = 2, D = 3, E = 5	A1 **(8)**
(b)	(£)156	B1 **(1)**
(c)	$29+29+27+x+33=156+5$ $x=43$	M1 A1 **(2)**
		11 marks

Notes for Question 3

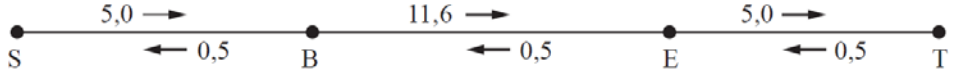
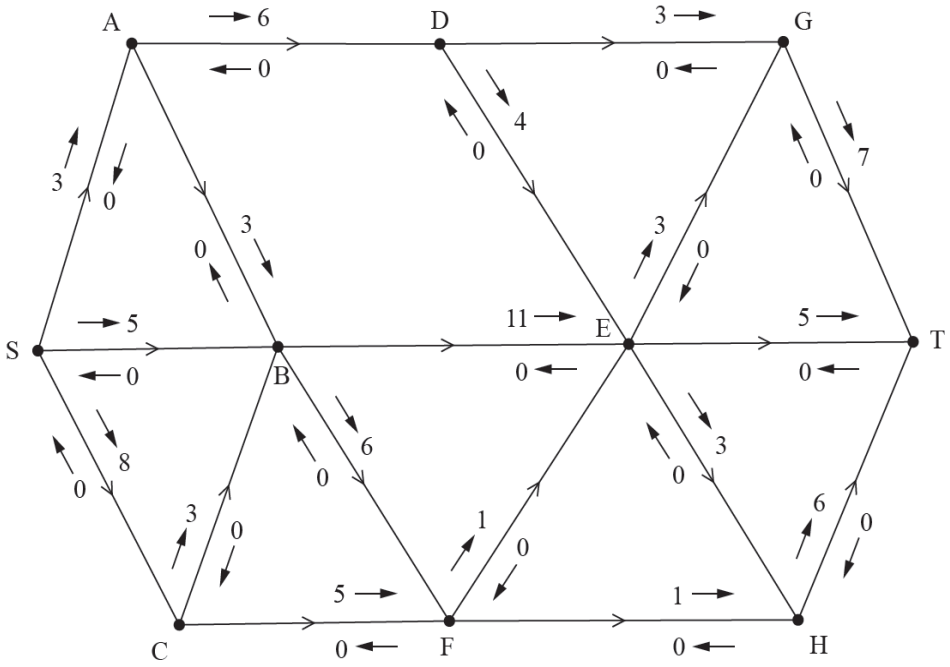
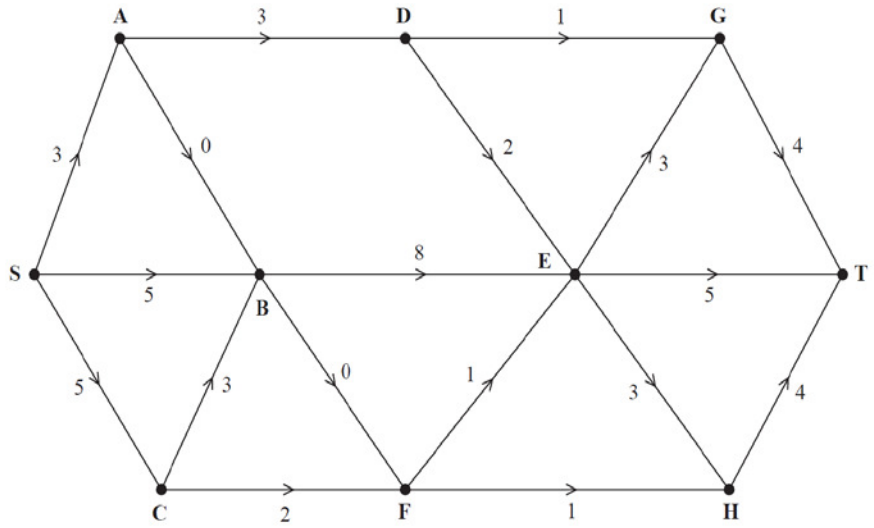
a1M1: Reducing rows and then columns (allow errors)
a1A1: CAO
a2M1: Double covered +e; one uncovered – e; and one single covered unchanged. 3 lines needed to 4 lines needed
a2A2: CAO
a3M1: One double covered +e; one uncovered – e; and one single covered unchanged. 4 lines needed to 5 lines needed (so getting to optimal table)
a3A1ft: Follow through on their previous table
a4A1: CSO on final table
a5A1: Correct allocation – dependent on all previous M marks – need not be stated but must be clear – allow if stated in (b)

b1B1: CAO – dependent on all previous M marks awarded in (a) – units not required

c1M1: Their allocation with A and D interchanged = their (b) + 5 (oe) - not dependent on any previous mark
c1A1: CAO

In (b) if a candidate replaces the x with '> 38' (or an equivalent general correct statement – but **not** a specific value greater than 38) and then after row reduction has '> 4' etc. then this can possibly score full marks (as this is equivalent to $x - 38$). See below SC2 and SC3 for when x is replaced by a specific value

SC1: If attempt to maximise this can score M1A0M1A0M1A0A0A0 (3 out of 8 max. in (a)) mark (b) and (c) according to main scheme
SC2: If x given a value greater than 38 this can score M1A0M1A0M1A1ftA0A1 (5 out of 8 max. in (a)) mark (b) and (c) according to main scheme
SC3: If x given a value less than or equal to 38 then M1A0M1A0M0A0A0A0 (2 max.)

Question Number	Scheme	Marks
4. (a)	Capacity of cut = 7 + 4 + 11 + 1 + 1 = 24	B1 **(1)**
(b)	The capacity of arc DG is 3 and the capacity of arc EG is also 3 and so the maximum that can flow in GT is 6 < 7 (the capacity of arc GT) and so GT cannot be full to capacity	B1 **(1)**
(c)	 Maximum flow through SBET is 5	B1 B1 **(2)**
(d)	e.g. SADEGT – 3 SADGT – 3 SADGT – 3 SABEGT - 3 SCFHT – 1 SCBEHT – 3 SCBEGT – 3 SCBADGT – 3 SCBEHT – 3 SCFHT – 1 SCFHT – 1 SCFHT – 1 SCFEDGT – 1 SCFEGT – 1 SCFEHT – 1 SCFEHT – 1 For reference: 	M1 A1 A1 A1 **(4)**
(e)	Max flow is 13 Cut through SA, SB, CB, BF, FE and FH (has a capacity of 13)	B1 B1 **(2)**
(f)	 e.g.	M1 A1 **(2)** **12 marks**

Notes for Question 4

a1B1: CAO (24)

b1B1: CAO – max. flow into G is 6, max flow out of G is 7 – answer must be numerical in nature (must contain 6 and 7)

c1B1: CAO (showing flow of 5 along SBET)
c2B1: CAO (maximum flow along SBET is 5)

d1M1: One valid flow augmenting route found and any value stated
d1A1: Two correct flow routes and values correct
d2A1: Three correct flow routes and values correct
d3A1: CSO flow increased by 8 and no more

e1B1: CAO (13)
e2B1: CAO (cut through SA, SB, CB, BF, FE, FH) - stated or shown

f1M1: Consistent flow pattern ≥ 11. Must have exactly one number on each arc
f1A1: CAO

For (d) as a guide SA must be increased by 3, SC by 5, CB by 3, nothing in SB, BF or ET, FE by 1 and FH by 1
For (f) as a guide SA = 3, SB = 5, SC = 5, CB = 3, CF = 2, BF = 0, FE = 1, FH = 1, ET = 5

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>5. (a)</td><td>$-2x-6y+z\le 40$
$2x+3y+2z\le 80$
$x+2y+2z\le 50$</td><td>M1
A1 (2)</td></tr>
<tr><td>(b)</td><td>
<table>
<tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>value</td><td>row ops</td></tr>
<tr><td>r</td><td>0</td><td>–3</td><td>3</td><td>1</td><td>1</td><td>0</td><td>120</td><td>R_1+2R_2</td></tr>
<tr><td>x</td><td>1</td><td>3/2</td><td>1</td><td>0</td><td>1/2</td><td>0</td><td>40</td><td>$0.5R_2$</td></tr>
<tr><td>t</td><td>0</td><td>1/2</td><td>1</td><td>0</td><td>–1/2</td><td>1</td><td>10</td><td>R_3-R_2</td></tr>
<tr><td>P</td><td>0</td><td>4</td><td>$4-k$</td><td>0</td><td>2</td><td>0</td><td>160</td><td>R_4+4R_2</td></tr>
</table>
</td><td>M1 A1
M1 A1 (4)</td></tr>
<tr><td>(c)</td><td>$4-k<0\ (\Rightarrow k>4)$</td><td>B1 (1)</td></tr>
<tr><td>(d)</td><td>
<table>
<tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>value</td><td>row ops</td></tr>
<tr><td>r</td><td>0</td><td>–9/2</td><td>0</td><td>1</td><td>5/2</td><td>–3</td><td>90</td><td>R_1-3R_3</td></tr>
<tr><td>x</td><td>1</td><td>1</td><td>0</td><td>0</td><td>1</td><td>–1</td><td>30</td><td>R_2-R_3</td></tr>
<tr><td>z</td><td>0</td><td>1/2</td><td>1</td><td>0</td><td>–1/2</td><td>1</td><td>10</td><td>R_3</td></tr>
<tr><td>P</td><td>0</td><td>$2+k/2$</td><td>0</td><td>0</td><td>$4-k/2$</td><td>$-4+k$</td><td>$120+10k$</td><td>$R_4-(4-k)R_3$</td></tr>
</table>
</td><td>B1 M1
A1 A1 (4)</td></tr>
<tr><td>(e)</td><td>$4-k/2\ge 0\ (\Rightarrow k\le 8)$</td><td>M1 A1 (2)</td></tr>
<tr><td>(f)</td><td>$(P=)120+10k$
$x=30,\ y=0,\ z=10,\ r=90,\ s=t=0$</td><td>B1
B1ft (2)</td></tr>
<tr><td>(g)</td><td>$160<P\le 200$</td><td>M1 A1 (2)</td></tr>
<tr><td></td><td></td><td>17 marks</td></tr>
</table>

Notes for Question 5

a1M1: Two correct equations (e.g. $-2x-6y+z+r=40$) or inequalities
a1A1: CAO (not equations)

b1M1: Correct pivot located (2 in x column), attempt to divide row. If choosing negative pivot M0M0
b1A1: CAO pivot row correct including change of b.v. (s must be changed to x)
b2M1: (ft) All values in one of the non-pivot rows correct **or** one of the non-zero/one columns correct (that is one of the y, z, s or value columns correct) following through their choice of positive pivot
b2A1: CAO on all values for first iteration – ignore row operations and b.v. column for this mark

c1B1: CAO (must be strict inequality)

d1B1: CAO pivot row correct including change of b.v.
d1M1: All values in one of the non-pivot rows correct or one of the non zero and one columns (y, s, t or value) correct following through their choice of pivot from column z
d1A1: Row operations used correctly at least twice, i.e. two of the non zero and one columns (y, s, t or value) correct
d2A1: CAO – both iterations - all values correct and all eight row operations correctly stated – allow if row operations given in terms of old row 2 – ignore b.v. columns for this mark

e1M1: Setting <u>any</u> of the expressions in terms of k from the P row from their second iteration $>$ or ≥ 0 - dependent on all M marks in (b) and (d) and both correct pivot rows (but ignore b.v. label)
e1A1: CAO (oe) – need not be simplified but must not be strict inequality – if no working shown and fully correct award both marks, if no working shown and incorrect M0 A0

f1B1: CAO
f2B1ft: Follow through their values - dependent on all M marks earned in (b) and (d)

g1M1: Either 160 or 200 seen (but not as a term in an equation)
g1A1: CAO

Question Number	Scheme					Marks
6.	Stage	State	Action	Dest.	Value	B1 **(1)**
	0	I	IS	S	30 – 5 = 25*	
		J	JS	S	29 – 3 = 26*	
	1	F	FI	I	35 – 5 + 25 = 55*	
			FJ	J	35 – 7 + 26 = 54	
		G	GI	I	36 – 5 + 25 = 56*	M1 A1 A1**(3)**
			GJ	J	36 – 7 + 26 = 55	
		H	HI	I	38 – 6 + 25 = 57*	
			HJ	J	38 – 7 + 26 = 57*	
	2	D	DF	F	24 – 7 + 55 = 72	
			DG	G	24 – 6 + 56 = 74*	
			DH	H	24 – 8 + 57 = 73	M1 A1ft A1 **(3)**
		E	EF	F	22 – 6 + 55 = 71	
			EG	G	22 – 6 + 56 = 72	
			EH	H	22 – 4 + 57 = 75*	
	3	A	AD	D	27 – 6 + 74 = 95	
			AE	E	27 – 4 + 75 = 98*	
		B	BD	D	29 – 5 + 74 = 98	
			BE	E	29 – 3 + 75 = 101*	M1 A1ft A1 **(3)**
		C	CD	D	32 – 6 + 74 = 100	
			CE	E	32 – 5 + 75 = 102*	
	4	S	SA	A	– 3 + 98 = 95	M1 A1 **(2)**
			SB	B	– 4 + 101 = 97*	
			SC	C	– 6 + 102 = 96	
	Optimal expected income: (£) 9700					B1
	Optimal schedules are: S – B – E – H – I – S S – B – E – H – J – S					B1B1 **(3)**
						15 marks

Notes for Question 6

- Condone lack of destination column
- Only penalise incorrect result in value – i.e. ignore working values
- Penalise absence of state or action column with first two A marks earned only
- Penalise empty/errors in stage column with first A mark earned only

For all M marks – must bring optimal results from previous stage into calculation at least once – so from Stage 0 if neither of 25 or their 26 are used in Stage 1 then M0. Ignore extra rows. Must have correct ingredients (appearance fee, cost) at least once per stage. If no working seen then values in the Value column must be correct to imply these marks

1B1: CAO (Stage 0)
1M1: First stage completed. At least six rows. Bod if something in each cell
1A1: Any two states correct with no extra rows for these states
2A1: All 3 states correct with no extra rows for Stage 1
2M1: Second stage completed. At least six rows. Bod if something in each cell
3A1ft: One state correct (ft) from Stage 1 (no extra rows)
4A1: Both states correct for Stage 2 (no extra rows)
3M1: Third stage completed. At least six rows. Bod if something in each column
5A1ft: Two states correct (ft) from Stage 2 (no extra rows)
6A1: All 3 states correct (no extra rows)
4M1: Fourth stage completed. At least three rows. Bod if something in each cell
7A1: Final state correct (no extra rows)
2B1: CAO (dependent on all previous M marks earned) – units not required – but not for 97
3B1: One route correct (dependent on all previous M marks earned) – condone no S's but not reversed
4B1: Both routes correct (dependent on all previous M marks earned) – no additional routes, S's present and not reversed

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